Fortnightly Problems

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Introduction

I posted these challenges on the UCL Maths society's (ADM's) department common room bulletin for students to solve in 2023/24 term 2. The idea of these questions is to be difficult, but for there to be an answer that fits on a post-it.

Challenge 1 (14/01/2024)

A knight move in chess is a translation 2 units North, East, South or West in one direction and 1 unit in a perpendicular direction. Given a knight starts at (0,0) in $\mathbb{Z}^2$, what is the minimum number of moves required for it to reach (n, n) for all $n \in \mathbb{N}$?

Challenge 2 (22/01/2024)

Let $\langle x_n \rangle = 5n$ for values n from 1 to $4N$ inclusive. How many subsequences sum to a multiple of 4?

Challenge 3 (19/02/2024)

Consider the right angled triangle with sides $a < b < c$ (and angles A and B opposite a and b respectively). Prove:

$$a = b \tan\left(\frac{A}{2}\right) + \frac{c - b}{\cos(B)}$$

Challenge 4 (Added Later)

A projectile is projected at an angle of θ and a speed of U. Prove that the following quadratic is tangential for all projectile paths for fixed U:

$$y = \frac{U^2}{2g} - \frac{gx^2}{2U^2}$$

The parametric equation for motion is $\mathbf{r} = \begin{pmatrix} Ut\cos(\theta) \\ Ut\sin(\theta) - \frac{1}{2}gt^2 \end{pmatrix}$

Challenge 5 (Added Later)

Let a 1D annulus be given by $D(p, r_1, r_2) := \{x \in \mathbb{R} : r_1 < |p - x| < r_2\}$. Prove that any set of $n - 1$ points on the real line can always be covered by a collection of non-intersecting 1D annuli of the form $D\left(p, \frac{1}{2n}, \frac{1}{2}\right)$ (i.e an interval of unit length with a hole of size n^{-1})